Joint Distribution of Stock Market Returns and Trading Volume

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ABSTRACT

Bivariate normal distribution is fitted to monthly stock market returns and trading volume data from Muscat Securities Market (MSM). Marginal and conditional distributions are derived. The graphs of the fitted distributions of both the joint and marginal distributions appear to be normal. The empirical joint probabilities are worked out which show a significant joint movement of the volume and returns. The results indicate that the joint probabilities are higher when the trading volume is close to monthly average and the returns are positive. The marginal probabilities show that the returns are positive with probability of 0.43 and it will occur on average with a return period of two months. In addition the conditional expectation of the returns indicates a significant correlation of volume with the returns.

Keywords: Bivariate normal distribution; Trading Volume; Stock market returns;

Marginal Distributions; Conditional distributions; conditional expectation.

1. INTRODUCTION

The relationship between trading volume and stock returns has been the main issue of theoretical and empirical research for a long time. The markets have to be aware about trading volume which reflects how the informed traders and uniformed traders interact with each other in the marketplace (Fauzia Mubarik & Attiya Y. Javid, 2009). This study attempts to explain the effect of trading volume on the probability of the stock returns of Muscat Security Market.

The nature of the stock return distribution has been very vigorously investigated in a univariate framework (Richard et. Al. ,2001; Kevin. S. 2013). Trading volume which is the total number/quantity of stock contracts sold during a trading day is a very strong indicator of the market activity. This might significantly affect the shape of the distribution of returns and consequently the probabilities of the return. Therefore relationship between the stock return and trading volume as determined by their joint distribution needs to be investigated. This study focus on the empirical joint distribution of trading volume and stock returns assuming that the bivariate normal distribution would adequately represents this joint process.

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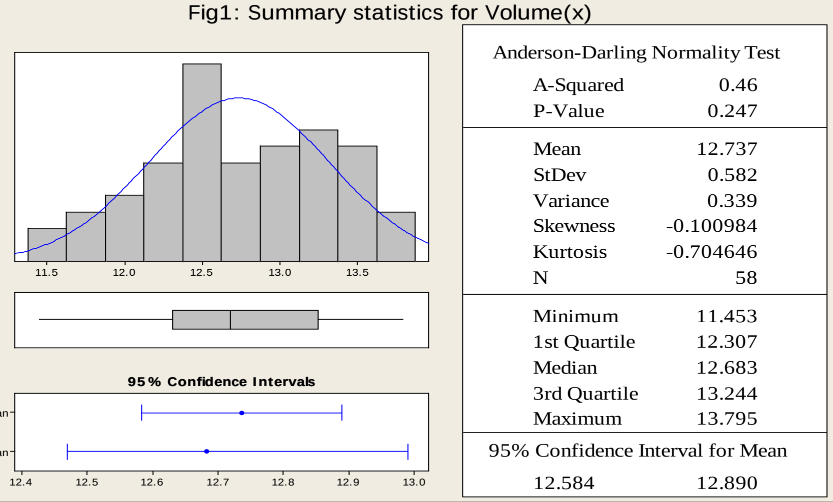
2. DATA AND DESCRIPTIVE STATISTICS

The Muscat Securities Market (MSM) was established by the Royal Decree (53/88) issued on21st June 1988 to regulate and control the Omani securities market and to participate, effectively, with other organizations for setting up the infrastructure of the Sultanate's financial sector. The main index of Muscat Securities Market has been established in 1992. A number of companies included in the index sample have changed overtime to reach currently 30 companies, the most liquid in the market.

Five years monthly data on the MSM index in the form of log differenced returns (y) and the trading volume (x) as measured by log of the number of shares traded was taken from the official website of the MSM. The trading volume could be measured either by number of stocks traded or by the value of the stocks traded. We have taken the number of shares as the trading volume, because the number of shares better shows the market activity. The summary statistics and the histograms of the univariate distributions of x and y are presented in figures 1 and 2. For the logarithm of the number of shares data, the mean is 12.737 with standard deviations 0.582.The distribution of these data is normal because the Anderson-Darling test of normality gives p-value >.05. For the returns data, the mean is 0.5769 with standard deviations 3.7053.The univariate distribution of these data is also normal because the Anderson-Darling normality test gives p-value >.05. The Pearson correlation of volume and return was calculated to be 0.366 which was highly significant at 1% level. This makes it necessary to investigate the joint distribution because the probabilities of returns based on marginal distribution of stock returns would be misleading in the presence of significant correlation.

Fig1: Summary statistics for Volume(x)



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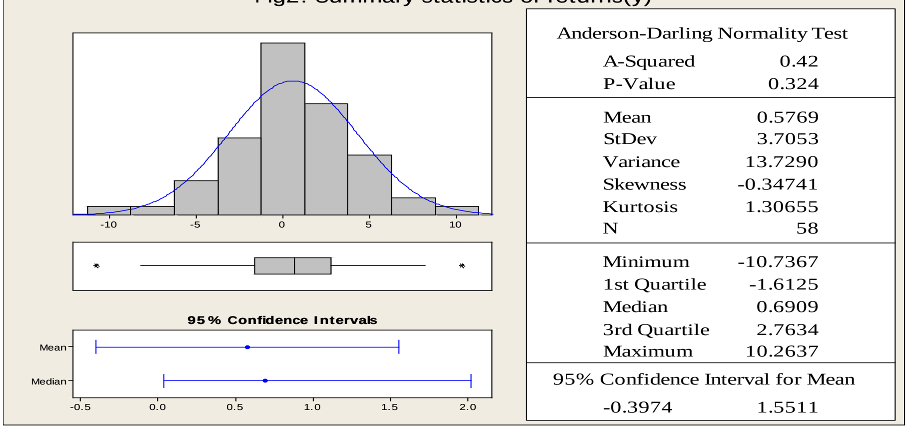

Fig2: Summary statistics of returns(y)

3. BIVARIATE NORMAL DISTRIBUTION

The bivariate normal distribution is used in this section to find the joint probability distribution of correlated returns of the share price (y) and trading volumes(x). The joint pdf, the marginal and conditional distributions are as below (Wilks, D. S. 2006).

$$f(x,y)=\frac{1}{2\pi\sqrt{\sigma_x^2\sigma_y^2(1-\rho^2)}}\exp\left\{-\left(\frac{1}{2(1-\rho^2)}\right)\left[\frac{(x-\mu_x)^2}{\sigma_x^2}-\frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}+\frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\} \quad -\infty<x,y<\infty \qquad \textbf{(1)}$$

Where:

$$\mu_x=E(x) \qquad \sigma_x^2=V(x)$$

$$\mu_y=E(y) \qquad \sigma_y^2=V(y)$$

$$\rho=\frac{E[(x-\mu_x)(y-\mu_y)]}{\sigma_x\sigma_y}$$

3.1 MARGINAL DISTRIBUTIONS

The marginal distributions are derived by integrating the joint distribution over other variable and are given as below:

$$f_x(x)=\frac{1}{\sqrt{2\pi\sigma_x^2}}\exp\left\{-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right\} \quad -\infty<x<\infty \qquad (2)$$

$$f_y(y)=\frac{1}{\sqrt{2\pi\sigma_y^2}}\exp\left\{-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right\} \quad -\infty<y<\infty \qquad (3)$$

$$x\sim N(\mu_x,\sigma_x^2) \qquad Y\sim N(\mu_y,\sigma_y^2)$$

3.2 CONDITIONAL DISTRIBUTIONS

The conditional probability distribution of Y given X can be derived by dividing the joint distribution of x and y with marginal distribution of x and is expressed as follows:

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$$f(y \mid x) = \frac{1}{\sqrt{2\pi\sigma_y^2(1-\rho^2)}} \exp\left\{-\left(\frac{1}{2(1-\rho^2)\sigma_y^2}\right)\left[y - \left(\mu_y + \frac{(y-\mu_x)\rho\sigma_y}{\sigma_x}\right)\right]^2\right\} \quad -\infty < y < \infty \qquad \textbf{(4)}$$

Where

$$Y \mid x = x \sim N\left[\mu_y + \frac{(x-\mu_x)\rho\sigma_y}{\sigma_x}, \sigma_y^2(1-\rho^2)\right]$$

Using these conditional distributions we can work out the conditional expectation as below:

$$E(y|x] = \int_{-\infty}^{\infty} yf(y|x)dy \qquad (5)$$

$$= \mu_y + \frac{\rho\sigma_y}{\sigma_x}(x - \mu_x)$$

$$var[y|x] = \sigma_y^2(1-\rho^2)$$

Then the conditional expectation is linear with $\beta_0 = \mu_y - \frac{\rho\sigma_y\mu_x}{\sigma_x}$ and $\beta_1 = \frac{\rho\sigma_y}{\sigma_x}$

This implies that the regression model would be: $\mathbf{y = \beta_0 + \beta_1 x + \epsilon}$ and

$$E(y|x] = \beta_0 + \beta_1 x$$

where

$$E(\epsilon) = 0 \quad , \quad var(\epsilon) = \sigma^2$$

This justifies the use of regression analysis when the volume as a regressor is also a random variable. (Koyed, A. 2008)

4. JOINT PROBABILITIES

The empirical joint probabilities are computed on the same principle of single variable. A bivariate frequency classification table was first constructed by taking suitable class limits of returns and volume. Then the joint class frequencies were counted and the frequencies in the limits of row *i* and column *j* of the table is defined as the joint empirical probability function of the two random variables and is estimated by (Yue, et al. **,**1990):

$$f(x_i, y_j) = \frac{n_{ij}}{N+0.12} \qquad (6)$$

Where *N* is the total number of observations, and n_{ij} is the number of occurrences of the combinations of x_i and y_j. The mid points of class limits are denoted by x_i and y_j. Theoretical joint probabilities are estimated using Eq. (1) by replacing parameters by their respective moment estimates and mid points by x_i and y_j.

The empirical joint distributions estimated from the bivariate table are presented in table (3) along with the bivariate frequencies. This table also contains the theoretical joint probability worked out using the bivariate normal distribution in the brackets. The

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marginal distribution of x and y for the observed data and the theoretical marginal distribution are also presented in this table.

This table shows that the theoretical probability is higher than the observe one for the marginal probability of the trading volume. In contrast, the theoretical probability is smaller than the observe one for the marginal probability of the return. We may need to

Table#1: Bivariate frequency, empirical joint probabilities along with theoretical bivariate normal probabilities. The numbers inside the brackets show the theoretical bivariate normal probability.

Mid x Mid y	11.3	11.8	12.3	12.8	13.3	13.8	Marginal probability
-8.95				2 0.0344 (0.0015)			0.0344 (0.003921)
-4.95	1 0.0172 (0.0033)	1 0.0172 (0.0135)	2 0.0344 (0.025456)	1 0.0172 (0.020370)	1 0.0172 (0.006951)		0.1032 (0.035163)
-0.95	1 0.0172 (0.003274)	4 0.0688 (0.021323)	8 0.1376 (0.0592112)	6 0.1032 (0.070120)	5 0.0860 (0.035412)	1 0.0172 (0.007627)	0.43 (0.098286)
3.05			9 0.1548 (0.0358511)	5 0.0860 (0.062830)	6 0.1032 (0.046959)	2 0.0344 (0.014967)	0.3784 (0.08563)
7.05					1 0.0172 (0.016210)	1 0.0172 (0.007646)	0.0344 (0.023253)
11.05					1 0.0172 (0.001456)		0.0172 (0.001968)
Marginal probability	0.0344 (0.032435)	0.086 (0.187273)	0.3268 (0.516636)	0.2408 (0.680996)	0.2408 (0.428899)	0.0688 (0.129068)	

choose an improved method of estimation of empirical distribution since the method we are using is biased.

5. CONDITIONAL PROBABILITIES

The conditional probability of returns given trading volume is estimated based on equation(4) and is shown in Table (4).These probabilities are estimated for selected values of volume.

Table#2 The conditional probability of return of share price index given trading volume

f(y|x=13.3)	f(y|x=12.3)	f(x,y)	y
0.071429	0.105263	0.0344	-4.95
0.357143	0.421053	0.1376	-0.95
0.428571	0.473684	0.1548	3.05

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We then examined the return(r)-volume(v) relationship in regression framework. Since the volume is also a random variable there we used the conditional expectation and the fitted regression results are are below: The estimated regression equation is

$$E(r_t \mid v) = -29.1 + 2.33\, V_t \qquad (7)$$

Predictor	Coef	SE Coef	T	P
Constant	-29.13	10.18	-2.86	0.006
V	2.3317	0.7987	2.92	0.005

S = 3.50970 R-Sq = 13.4% R-Sq(adj) = 11.8%

It is clear from this equation that the expected returns would be negative when the volume is low (Timothy J. Brailsford, 1994).

6. CONCLUSION

This study investigates the relationship between stock returns and trading volume based on the monthly data of Muscat Security Market from 2009 to 2013. The empirical results verify that there is a significant interaction between trading volume and returns. The results indicate that the joint probabilities are higher when the return is positive and trading volume is close to the average. That is the return is higher when the trading volume is between about 20 to 60 million. The marginal probabilities show that the returns are positive with probability of 0.43 and it will occur on average after two months. In addition, the cumulative probabilities indicate the returns are positive with higher probabilities when the trading volume is higher than 60 million, and it would occur on average after 3 months.

Various regression models were fitted to investigate the relationship between returns and the trading volume. All these models showed that the trading volume affects the return significantly. However the coefficient of determinations was not very high. This may be due to some other problems of regression such as stationarity and autocorrelation which needs to be further investigated.

REFERENCES

[1] Ayinde, Kayode. (2008). Performances of Some Estimators of Linear Model when Stochastic Regressors are Correlated with Autocorrelated Error Terms. European Journal of Scientific Research;May2008, Vol. 20 Issue 3, p558.

[2] Fauzia Mubarik & Attiya Y. Javid.(2009). Relationship between Stock Returns, Trading Volume and Volatility Evidence From Pakistani Stock Market. *Asia Pacific Journal of Finance and Banking Research. Vol. 3. No. 3*. pp1-17.

[3] Kevin. S. (2013) A Study of the Relation Between Market Index, Index Futures and Index ETFs: A Case Study of India . Rev. Integr. Bus. Econ. Res. Vol 2(1).

[4] Richard D. F. Harris and Coskun C. K. (2001) The Empirical Distribution of UK and US Stock Returns. Journal of Business Finance & Accounting, 28(5) & (6), June/July 2001, 0306-686X.

ISSN: 2304-1013 (Online); 2304-1269 (CDROM); 2414-6722 (Print)

[5] Timothy J. Brailsford.(1994). The empirical relationship between the trading volume, return and volatility.
[6] Wilks D. S. (2006). Statistical Methods in Atmospheric Sciences. Elsevier Inc. UK.
[7] Yue, S. Ouarda, T. Bobe,B. Legendre,P Bruneau, P. (1990). The Gubmel Mixed Model for flood frequency analysis**.** Journal of Hydrology**.**226,88-100.

ISSN: 2304-1013 (Online); 2304-1269 (CDROM); 2414-6722 (Print)