Spying Solution In The Framework Of Terrorist Conflicts

Sylvain Baumann
Normandie University, France

ABSTRACT

Common knowledge is not always available for all kinds of games. When this information is reachable for players, experimental evidences consider that the Nash Solution is not played by humans: all players are not rational. It leads to a behavioral equilibrium which depends on this private information. The aim of this paper is to introduce new concepts of solution and to explain the interest of the spying in the framework of terrorist conflicts.

Keywords: Non-cooperative game, altruism, terrorist conflict, spying game

1. INTRODUCTION

Game theory, founded by von Neumann and Morgenstern (1944), enables to analyze the conflict situations. When a player has to choose a strategy, the Nash solution (1950) will usually be applied to every finite game : *"Nash equilibrium is so widely accepted that the reader can assume that if a model does not specify which equilibrium concept is being used, it is Nash equilibrium"* (Rasmussen, 2007). In static games of complete information, the Nash solution is determined when each player's predicted strategy is this player's best response to the predicted strategies of the other players. In fact, this solution means that no player wants to deviate.

Classical game theory is based on strict hypothesis: players can have imperfect information about the game rule and the characteristics of the players. Harsanyi (1967) developed the bayesian game to fill this lack of information. However, for Nash or bayesian solutions, it can lead to a non-optimal solution (Osborne and Rubinstein, 1994 ; Kahan and Goehring, 2007). In the case where all the information is available, the Nash solution is not always suitable in the reality. Indeed, experimental evidences show that the players can reach a Pareto superior issue (Colman, 1995; Ledyard, 1995; Sally, 1995).
What about multiple equilibria? What about the behavior rule of each agent? The issue can be different according to the environments of each agent: their rationality, their moral principles, their risk aversion... The game rule of expected utility meets some criticism: Allais paradox (1953), Ellsberg paradox (1961) where players don't

respect the postulates of subjective expected utility, Kahneman-Tversky's prospect theory (1992).

New solutions were developed taking into account of these criticisms: the evolutionary games (Maynard Smith, 1982), the Berge equilibrium (Berge, 1957; Colman, 2011), the cooperative equilibrium (Capraro, 2013). These two last equilibria underline the altruism of players. In the Berge equilibrium, players have to choose the strategy which maximizes the welfare of the others when the other players do the same thing (Courtois, 2015). Concerning the cooperative equilibrium, this new solution differs from the usual solutions because it considers the cooperation and the cumulative prospect theory instead of expected utility theory.

Some authors establish a correlation between altruism and suicide attack (Kamas, 2005). The principal achievement of this paper is to formalize new concepts of solutions in the framework of the terrorist conflicts through game theory. The structure of the paper is as follows. In Section 2, we call back the criticism about the Nash solution and the game rule. In Section 3 we give new kind of solutions to explain some human behaviors. Section 4 introduces the concept of behavioral equilibria. In Section 5 we focus our analysis on the concept of spying game and we study the decision-making process in order to determine the spying solution. The last Section puts forward some conclusions.

2. BETWEEN UNCERTAINTY AND BEHAVIOR: WHY USE SPYING GAMES?

Before determining the equilibrium, we have to consider some parameters, such as the player behavior and the environment in which the game takes place. Usually, we use the Nash solution but all the players are not rational. Some situations can lead to indetermination. This section calls back criticism about Nash solution and here we define new concepts.

2.1. Uncertainty on the Nash equilibrium with mixed strategies

A mixed strategy is a probability distribution on the pure strategies. It is a random choice of a pure strategy (Nash, 1950). For Harsanyi (1973), it represents a player's uncertainty about the choice of pure strategy for other players.

Recall that in the Matching Pennies (Table 1) there is no pure strategy Nash equilibrium. Suppose that p and *(1-p)* respectively designate the probability of playing A and B for the player 1 (q and *(1-q)* for player 2).

	Player 2	
	A	*B*
A	1 / -1	-1 / 1
B	-1 / 1	1 / -1

Table 1 – Matching Pennies

($p=1/2$; $q=1/2$) is a mixed strategy Nash equilibrium. What about the decision of each agent? They are indifferent between their strategies. Their beliefs (Harsanyi, 1973) don't let them to be sure to have a positive payoff. This criticism about the mixed strategies is not recent. This is why some modelers give priority to pure strategies.

2.2. Uncertainty about the player's behavior

Recall that some experimental evidences show that some human agents don't play the Nash solution. We have to consider cautious strategies. For example, some players will choose a strategy which gives them a positive payoff. The use of the strategy *Maximin* is a good option to risk-averse player. Take the example in Table 2.

The Nash equilibrium of this game is given by the issue *(B,R)* corresponding to the payoff *(1,20)*. However Player 1 has uncertainty about the rationality of Player 2. Player 1 can lose 50 units, whereas he can only win 1 in the case of the Nash equilibrium. This risk-averse player chooses his cautious strategy: he maximizes his minimum payoff and plays *H*. The new equilibrium is *(H,R)* supposing that Player 2 is rational and thinking that Player 1 is rational too.

		Player 2			
		L	M	R	Min J_1
	H	0	5	0	
		5	0	0	0
	C	10	-2	-1	
		1	-1	0	-1
	B	19	15	20	
		-50	2	1	-50

Table 2 – Risk-averse Player 1

3. NEW BEHAVIORAL SOLUTIONS TO THE PROBLEM OF TERRORISM

3.1. Superiority solution

Consider now that Player 1 has another kind of behavior. He wants to get a greater payoff than his opponent. A superiority solution for a player i is defined as follows:

Definition: Superiority Solution
A feasible strategy profile (s_i^*, s_{-i}^*) is said a superior solution for a player i included in N, if and only if: $U_i(s_i^*, s_{-i}^*) > U_j(s_i^*, s_{-i}^*)$, where $i \neq j$ and $j \in N$

Supposing that Player 2 is rational, he will choose the strategy R corresponding to the Nash equilibrium. Player 1 plays C in order to get 0 greater than -1 (payoff of Player 2).

3.2. Sacrifice solution

After defining a selfish equilibrium, we can define, at the opposite, an altruistic equilibrium. Some authors developed this concept. In the case of the Berge equilibrium, players have to choose the strategy which maximizes the welfare of the others when the other players do the same thing. The cooperative equilibrium is altruistic because of a coalition which maximizes the payoff of this coalition. What about a coalition where a player wants to maximize the payoff of his partner(s) regardless of his payoff? This situation fits with the terrorist conflict.

Example:

We consider a three-player game with respectively two endogenous players and a exogenous player: a terrorist group *(T)*, a potential suicide bomber *(B)* and the family of this potential terrorist *(F)*. The objective of the group is to recruit a person able to become a human weapon. This person has the choice between two strategies: either he accepts to become a terrorist, or he tries to find a legal job. In this poor country, he has to support his family with his income. If he finds a legal job, the wage is not enough to feed his family. The terrorist group offers this bargain: *"If you join to us and accept to throw a suicide attack, everyone will hold you in high regard. You will be seen as a hero. In reward, we pay an income to your family members so as to support them."*

To simplify this game and in order to explain the new concept solution, we suppose a one-shot game where the utility functions don't take into account the inter-temporality. If the agent B accepts the deal, his family gets a fund R. However he will lose his life. The value of his life is denoted by V_B. The suicide attack causes damages for an amount of D to the targeted country. If he decides to work in a legal job, he earns the income L. To make credible this game, we have the following hypotheses:

$$D > R$$
$$R > L$$

The first hypothesis points out the fact that the terrorist group utility is higher when it attacks. The second underlines that the fund from the terrorist is higher than the income from a legal work in a poor country.

The table 3 resumes the payoff of each agent according to their strategies. We suppose that the family has no strategy because this player is exogenous. This is why we can resume this game in a *2×2* matrix.

The payoffs in this matrix are given by (Potential terrorist, Terrorist group, Family). The Nash equilibrium of this game is (Legal job, Attack, .).

| | Terrorist group | |
	Attack	No attack
Join terrorists	R $D-R$ $-V_B$	R 0 0
Legal job	L 0 L	L 0 L

Table 3 – Suicide Attack: sacrifice solution

Definition: Strictly Sacrifice Solution

Consider a coalition $C \subseteq N$ with $i \in C$. A feasible strategy profile $s^* \in S$ is said a strictly sacrifice solution for:

$$Max\ U_{C-\{i\}}(s^*)$$

$$with\ U_{C-\{i\}} = \sum_{j \in C} U_j\ \ where\ i \neq j$$

Definition: Weakly Sacrifice Solution

Consider a coalition $C \subseteq N$ with $i \in C$. A feasible strategy profile $s^* \in S$ is said a weakly sacrifice solution for:

$$Max\ U_{C-\{i\}}(s^*)$$

From this example and the definition 2, we can determinate the strictly sacrifice solution for the potential terrorist. He forms a "coalition" with his family: C={B,F}. Knowing that the terrorist group decides to attack and the family has only one strategy, the potential terrorist has to decide a strategy in order to maximize the payoff of the family:

$$max\ U_F=max\{R,L\}=R$$

The weakly sacrifice solution of the potential terrorist B to the coalition C is (Join terrorists, Attack, .).

4. ANALYSIS OF BEHAVIORAL STRATEGIES

4.1.Behavioral strategies

We assume that some players can be irrational. In this paper we only consider these kinds of behavior: rational, altruist, risk-averse, cooperative. Players have an optimal strategy for each kind of behavior. We resume it in the table 4.

Behaviors	Notations	Strategy
Rational	Rat	Nash equilibrium
Altruist	Alt	Sacrifice solution
Risk-averse	$Risk$	Maximin strategy
Cooperative	$Coop$	Berge equilibrium

Table 4 – Behavioral Strategies

A rational player uses the Nash equilibrium. For altruistic players, there are two possibilities: either he is totally altruist and plays the sacrifice solution, or his altruism leads to maximize the payoff of the other players knowing that the other players do the same (Berge equilibrium). A risk-averse behavior implies using the *Maximin* strategy.

Definition: Nash Equilibrium
A feasible strategy profile $s^* = (s_i^*)_{i \in N} \in S$ is a Nash equilibrium if and only if, $\forall i \in N, \forall s_i \in S_i$:
$$U_i(s_i^*, s_{-i}^*) > U_i(s_i, s_{-i}^*)$$

Definition: Maximin Strategy
A player looks at the minimum payoff that he can get for each strategy. Then he chooses the strategy who gives him the maximum between these minimum payoffs: it is the *Maximin* solution.

Definition: Berge Equilibrium
A feasible strategy profile $s^* = (s_i^*)_{i \in N} \in S$ is a Berge equilibrium if and only if, $\forall i \in N, \forall s_{-i} \in S_{-i}$
$$U_i(s_i^*, s_{-i}) \leq U_i(s^*)$$

4.2.Behavioral equilibria

Let us consider the three player game of the Table 5, in which Player 1 chooses the row, Player 2 chooses the column and Player 3 chooses the matrix.

	Player 2	
	A	B
a	2 3 4	0 1 2
b	5 4 1	9 2 6

Matrix α

	Player 2	
	A	B
a	1 9 3	3 6 8
b	1 0 0	2 1 2

Matrix β

Table 5 – Three-person game

Usually we suppose that all the players are rational. Consequently we determine the Nash equilibrium. However we explain that several behaviors can be present in the game. By default, we make the hypothesis that all players suppose that the other players are rational.

When all players are rational, the equilibrium of this game is *(a,A,α)* corresponding to the Nash equilibrium. When the players are altruist and cooperative, the Berge equilibrium corresponds to the issue *(a,B, β)*. If Players are risk-averse, they maximize their minimum payoffs between their strategies. Player 1 chooses the strategy *a* (he has to *max (2,0)=2*), player 2 chooses B (*max (0,1)=1*) and the player 3 plays *β* (*max (0,1)=1*). Concerning the sacrifice solution, it depends on the coalition. The player 1 decides to play *b* if he wants to maximize only the payoff of the other players: *max(U₂+U₃)=max(3+2;4+5)=9*. In this case, player 1 supposes that the other players are rational.

In this example, there are three players and four behaviors. It gives us *4³=64* feasible situations. Let us take four situations giving us a different equilibrium in each case:

- *Coop vs. Rat vs. Risk: (a,A,β)*
- *Coop vs. Risk vs. Coop: (a,B,α)*
- *Rat vs. Risk vs. Coop: (a,B,β)*

- *Alt vs. Coop vs. Rat: (b,B,α)*

This example shows that it is difficult for players to predict the final issue. A player knowing the behavior of opponents can change his strategy *ceteris paribus*: a new equilibrium results from the spy. We have to define the concept of the spying game.

5. CHARACTERIZATION OF SPYING GAMES

Previous section underlines that players have different behaviors: risk-averse, altruist, selfish... How to decide of a strategy without knowing the behavior of other opponents? There are several kinds of spying:
- A player spies to get more information: the behavior of other players, the other player's strategy.
- A player spies another player, who detects the spying attempt. Either the spying player knows that the attempt fails or he is ignorant of this failure.
- Spying and counter-intelligence.
- In dynamic games, a player can indirectly have information on the player's behavior by analyzing the history of previous actions.

In this paper we only focus on the behavior spying.

5.1. Definitions

Spying games are composed of players who may have different behaviors and some of them have the possibility to spy other player(s). Here we suppose a static game. Before determining the solutions, we define this kind of games.

Definition:
A strategic form spying game Υ is given by:
$$\Upsilon = (N, K, M, O, (B_i)_{i \in N}, (S_i)_{i \in N}, (U_i)_{i \in N})$$
Such that:
- K is a finite set of spying players, where $K \subseteq N$;
- M is a finite set of myopic players, where $M \subseteq N$;
- O is a finite set of players who don't spy and are not myopic, where $O \subseteq N$;

- $N = \{1, ..., n\}$ is a finite set of players, with $N = (K \cup M \cup O)$ and $|N| = n = |K| + |M| + |O| - |M \cap K|$[1];
- (B_i) is the behavior for player i ; $B_i = \{Alt; Coop; Rat; Risk\}$;
- (S_i) is the set of available actions for player i;
- (U_i) is the payoff utility function for player i.

Definition:

A spy of a game Υ is a player having private information only on a set of myopic players. This private information embodies the behavior of these players.

Definition:

A myopic player of a game Υ designated a player whose information on his behavior are known by the spies. He acts as if this set of information is always private because he is ignorant of this spying.

Over a first phase we suppose that spying enables to have information on the player's behavior.

5.2.Spying player's behavior

Let us consider a three-player game in which Player 1 is a potential terrorist who concurs with the doctrine of a terrorist group (Player 2) and Player 3 is the targeted government. The potential terrorist can act in the name of the terrorist group even if this group ignores this fact. Table 6 resumes this game.

	Terrorist Group	
	C	D
A	−3 2 3	0 9 1
B	0 1 2	1 3 4

Potential Terrorist (row label)

Government strategy: E

---

[1] A player can be a spy and a myopic player at the same time.

Terrorist Group

		C	D
Potential Terrorist	A	5 1 4	−2 2 2
	B	1 0 2	−1 5 3

Government strategy: F

Table 6 – Three-person game

The first goal of this example is not to describe the strategies of each player (and their payoffs) but only to analyze their behavior. According to the player, his behavior is not the same:

- The potential terrorist agrees with the terrorist group ideology: this person sees himself as "altruist". He will choose a strategy who maximizes the payoff of the coalition {Potential Terrorist ; Terrorist Group} [2]: $B_1=\{Alt/C=\{1,2\}\}$.
- It is obvious that there are several kinds of terrorist groups. Some of them are rational; others want to inflict the maximum of damages. In our example, we consider a rational terrorist group: $B_2=\{Rat\}$.
- To counter the terrorist attacks, some governments choose defensive measures and others prefer to use preemptive strategies. The government wants to protect both the citizens and the infrastructures in order to guarantee a secure feeling and to limit the losses. We make the hypothesis that a government can be risk-averse: $B_3=\{Risk\}$.

After describing the behavior of each player, we can determine the behavioral equilibrium of this game. Then, we compare the solution with the spying case.

Without spying, we apply the behavioral equilibrium. We know that each player consider the other players as rational. Considering this information, the Nash equilibrium is given by: *(B,D,E)*. In fact, it corresponds to the player's belief on the strategy of the other players.

Which strategy will be chosen by players?

---

[2] In the case of a terrorist suicide bomber, he only maximizes the payoff of the other members of the coalition.

- The potential terrorist is altruist towards the group. He will maximizes the sum of their payoff knowing the expected strategies of players 2 and 3: *max {1+9 ; 4+3}=10*. The potential terrorist chooses the strategy *A* instead of the strategy *B*.
- The terrorist group is rational. So he decides to play *D*.
- The government is risk-averse, he maximizes its minimum payoff: *max {-3;-2}=-2*. The government plays *F*.

The behavioral equilibrium is given by *{A;D;F}* corresponding to the payoffs *(2;2;-2)*. In this example, we see that each player gets less than in the Nash equilibrium.

In order to choose the best strategy, players can use the spying to have information about the behavior of other players. We take back this example in which the potential terrorist spies the government. The terrorist group and the government don't spy. We get this following framework:

- Player 1 spies Player 3: Player 1 has knowledge on the Player 3's behavior.
- Player 3 is ignorant of this spy.
- Player 2 is a normal player: he does not spy and he is not spied.

Notation:

The set of spying players is *K={1}*.

The set of myopic players is *M={3}*.

The set of normal players is *O={2}*.

"*Player 1 spies Player 3*" is noticed: $1 \ll 3$.

Spying equilibrium:

We know that the players 2 and 3 keep the same strategies than in the behavioral equilibrium, i.e. $s_2=D$ and $s_3=F$. Player 1 knows that the Player 3 is risk-averse. So he deduces the strategy of the Player 3. Then, he maximizes the sum of the coalition payoffs: *max {2+2;3+5}= 8*. Player 1 plays $s_1=B$.

The spying equilibrium is given by *{B;D;F}* corresponding to the payoffs *(3;5;-1)*.

The aim of the spying is to have information on the behavior in order to choose the strategy which improves the payoff of a player or a coalition. In this example, Player 1 who spies the Player 3 enables to get a higher payoff for him and the Player 2. Consequently, the collective payoff is greater.

6. CONCLUSION

The purpose of this paper was to introduce a new kind of non-cooperative game: the spying game. With the open data world, it becomes easier to access to the private information. Lots of these information spread through the social networks and more generally through Internet. People mistakenly thought that their data were protected. Naturally, some cases exposed to the light of day, especially the Snowden Case. People were offended, but until now they continue to use the social networks. It is obvious that all the governments use this way for the safety of the nation. Knowing the behavior of a potential terrorist allows the government to take preventive measures. Our paper models these situations. However, to make this kind of game more realistic, we have to pursue our analysis by adding some situations: the counter-intelligence and the detection of a spying attempt.

REFERENCES

[1]   Allais, M. (1953), "Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine", *Econometrica*, 21, 4, 503-546.
[2]   Aumann, R.J. (1991), "Backward Induction and Common Knowledge of Rationality", *Games and Economics Behavior*, 8, 6-19.
[3]   Berge, C. (1957), "Théorie Générale des Jeux à n Personnes", *Gauthier Villars*.
[4]   Berge, C. (1957), "Topological Games with Perfect Information", *In Contributions to the Theory of Games*, Volume III, Edition M. Dresher, A.W. Tucker and P. Wolfe, Princeton University Press, 165-178.
[5]   Capraro, V., Venanzi, M., Polukarov, M., Jennings, N.R. (2013), "Cooperative equilibria in iterated social dilemmas", *In: Proceedings of the 6th International Symposium on Algorithmic Game Theory, Lecture Notes in Computer Science 8146*, 146-158.
[6]   Colman, A.M. (1995), "Game Theory and its Application in the Social and Biological Sciences", *Butterworth-Heinemann, Oxford (Second Edition)*.
[7]   Colman, A.M., Körner, T.W., Musy, O., Tazdaït, T. (2011), "Mutual Support in Games: Some Properties of Berge Equilibria", *Journal of Mathematical Psychology*, 55, 166-175.
[8]   Courtois, P., Nessah, R., Tazdaït, T. (2015), "How to play Games? Nash versus Berge Behaviour Rules", *Economics and Philosophy,* 31, 123-139.
[9]   Ellsberg, D. (1961), "Risk, Ambiguity and the Savage Axioms", *Quarterly Journal of Economics,* 75, 4, 643-669.
[10]   Fudenberg D., Tirole, J. (1991) "Game Theory", *The MIT Press, Cambridge*, *Massachusetts*.
[11]   Harsanyi, J. (1967), "Games with Incomplete Information Played by Bayesian Players", *Parts I, II, and III, Management Science*, 14, 159-82, 320-34, 486-502.

[12] Kahan, J.P., Goehring, D.J. (1990), "Responsiveness in Two-person Zero-sum Games", *Behavioral Science*, 18, 27-33.

[13] Kamas, L., Baum, S., Preston, A. (2005), "Altruistic Responses to the September 11 Terrorist Attacks: some Evidence from Dictator Games", *Eastern Economic Journal*, 31, 551-562.

[14] Kreps, D.M. (1990), "Game Theory and Economic Modelling", *Oxford University Press, Oxford*.

[15] Ledyard, J.O. (1995), "Public Goods: A Survey of Experimental Research", *Handbook of Experimental Economics, Princeton, NJ: Princeton UP*.

[16] Maynard Smith, J. (1982), "Evolution and the Theory of Games", *Cambridge University Press, Cambridge*.

[17] Nash, J. (1950), "Equilibrium Points in n-Person Games", *Proceedings of the National Academy of Sciences*, 36, 48-49.

[18] Osborne, M.J., Rubinstein, A. (1994), "A Course in Games Theory", *Cambridge, MA: MIT Press*.

[19] Rasmusen, E. (2007), "Games and Information", *Oxford, Basil Blackwell (Fourth Edition)*.

[20] Sally, D. (1995), "Conversations and Cooperation in Social Dilemmas: A Meta-Analysis of Experiments from 1958 to 1992", *Rationality and Society*,7, 58-92.

[21] Samet, D. (1996), "Hypothetical Knowledge and Games with Perfect Information", *Games and Economic Behavior*, 17, 230-251.

[22] Tversky, A., Kahneman, D. (1992), "Advances in prospect theory: Cumulative representation of uncertainty", *Journal of Risk and Uncertainty*, 5, 4, 297-323.

[23] von Neumann, J., Morgenstern, O. (1944), "Theory of Games and Economic Behavior", *2d ed. Princeton: Princeton University Press*.

[24] Spence, A.M. (1973), "Job Market Signaling", *Quarterly Journal of Economics*, 87, 355-74.